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STABILIZATION OF CONVECTIVE FLOW IN A VERTICAL LAYER USING A PERMEABLE PARTITION

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INTRODUCTION

The control of the stability of convective motions is one of the problems of applied hydrodynamics, since a loss of stability leads to a lowering to the characteristics of a number of technical objects (thermodiffusion columns, vertical heat-insulating layers, etc.). Some methods for the stabilization of convective flows are discussed in [1].

In the present article an investigation is made of the effect of a thin permeable partition, located at the interface between counterflows, on the stability of convective flow. A special characteristic of this means of stabilization is that a permeable partition, preventing the development of secondary motions, in practice changes the profile of steady-state flow and processes of molcular transfer. The effect of a permeable partition on the stability of a horizontal layer of liquid heated from below and of isothermal flow with a cubic velocity profile was investigated earlier in [2, 3].

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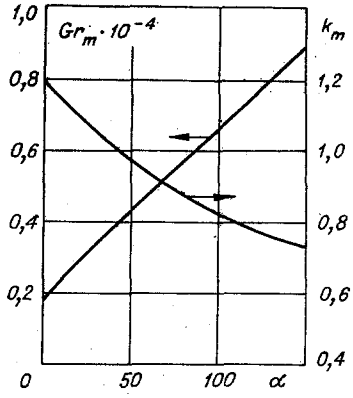

Fig. 1

1. Statement of Problem

We consider a vertical layer of liquid bounded by the solid surfaces $x = \pm h$ and having a temperature $\pm\Theta$. As is well known, in the layer there arises a steady-state convective flow with a cubic profile of the velocity $v_0(x)$, which becomes unstable for a sufficiently great temperature difference.

Let us investigate the effect on the stability of steady-state flow of a thin flat permeable partition located at the middle of the layer ($x = 0$) parallel to the bounding planes. Since for $x = 0$ the profile of the velocity $v_0(x)$ has a node, such a location of the partition does not change the steady-state distribution of the velocity and the temperature in the layer.

The amplitudes of the flat normal perturbations of the stream function $\varphi(x)$ and the temperature $\vartheta(x)$ satisfy the equations [1]

$$\begin{gathered}\varphi^{\mathrm{IV}} - 2k^2\varphi'' + k^4\varphi + ik\,\mathrm{Gr}\left[v_0''\varphi - v_0(\varphi'' - k^2\varphi)\right] + \vartheta' = -\lambda(\varphi'' - k^2\varphi),\\ \frac{1}{\mathrm{Pr}}(\vartheta'' - k^2\vartheta) + ik\mathrm{Gr}\left(T_0'\varphi - v_0\vartheta\right) = -\lambda\vartheta,\end{gathered} \tag{1.1}$$

where k and λ are the wave number and the complex decrement of the perturbations; Gr and Pr are the Grashof and Prandtl numbers; and $v_0 = (x - x^3)/6$ and $T_0 = x$ are the profiles of the velocity and the temperature of the steady-state flow. As the units of distance, time, velocity, and temperature in (1.1), h, h^2/ν, $g\beta\Theta h^2/\nu$, and Θ (ν is the kinematic viscosity; g is the acceleration of gravity, and β is the coefficient of thermal expansion are taken, respectively.

The vanishing of the perturbations of the velocity and temperature at the boundaries of the layer leads to the conditions

$$\varphi = \varphi' = \vartheta = 0 \quad (x = \pm 1). \tag{1.2}$$

With the statement of the boundary conditions at a thin permeable partition we postulate that, at the partition, the conditions of continuity are satisfied for the temperature, heat flux, and transverse component and that the longitudinal component of the velocity reverts to zero:

$$\vartheta_- = \vartheta_+, \frac{\partial\vartheta_-}{\partial x} = \frac{\partial\vartheta_+}{\partial x}, \quad \varphi_- = \varphi_+, \varphi_-' = \varphi_+' = 0 \quad (x = 0), \tag{1.3}$$

where the subscripts "–" and "+" denote, respectively, the values of the function to the left and right of the partition.

Due to the resistance of the partition to the flow of the liquid from one part of the layer to the other, there is the possibility of a pressure drop at the partition. We assume that the rate of suction of the liquid through the partition is proportional to this pressure drop:

$$v_x = -\alpha_1^{-1}(p_+ - p_-) \qquad (x = 0),$$

where α_1 is the resistance coefficient of the partition. Eliminating the pressure from this condition by means of the Navier–Stokes equation, for the stream function of the perturbations we obtain

$$\varphi_+''' - \varphi_-''' + k^2\alpha\varphi_+ = 0 \qquad (x = 0).$$

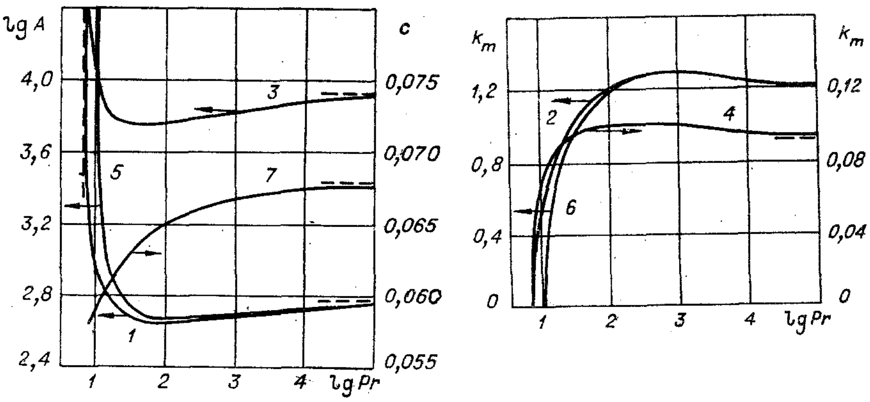

Fig. 2

Here α is the dimensionless resistance coefficient; as the unit of the change in the resistance we take η/h (η is the dynamic viscosity of the liquid).

The boundary-value problem (1.1)-(1.4) determines the spectrum of the decrements of the perturbations of convective flow in a vertical layer with a permeable partition.

Equations (1.1) were integrated from the boundaries of the layer to the partition by the Runge—Kutta method, with orthogonalization of three linearly independent solutions in each stage of the integration [4]. From the conditions of the joining of the solutions at the permeable partition (1.3) and (1.4), the spectrum of the decrement $\lambda = \lambda(k, Gr, Pr, \alpha)$ was determined.

2. Monotonic Instability

In the absence of a partition, convective flow in a vertical layer is unstable with respect to perturbations of two types, i.e., monotonic ($\lambda_i = 0$) and vibrational ($\lambda_i \neq 0$) [1]. Instability with respect to monotinic perturbations has a hydrodynamic character and leads to the formation of a system of steady-state eddies at the interface between the opposing flows. The minimal critical Grashof number, determining the limit of instability with respect to monotonic perturbations, depends only slightly on the Prandtl number, for small value of Pr, this type of instability is the main one.

A thin permeable partition located at the middle of the layer stabilizes the flow with respect to monotonic perturbations, since, at the partition, the longitudinal component of the velocity reverts to zero and the resistance of the partition hinders the development of closed flows. Calculations of the critical Grashof numbers were made for Pr = 0.01.

Figure 1 shows the dependence of the minimal critical Grashof number Gr_m and the wave number k_m of the most dangerous perturbation on the resistance of the partition. A calculation shows that, in the case of an absolutely permeable partition ($\alpha = 0$), $Gr_m = 1680$, which is more than three times as great as the critical Grashof number without a partition. With an increase in the resistance of the partition, there is an almost linear rise in the value of Gr_m. With a rise in the value of α, the wave number of the critical perturbations decreases monotonically.

3. Vibrational Instability

As is shown in [5], convective flow in a vertical layer, starting with $Pr_* = 11.4$, is unstable with respect to perturbations of the type of traveling waves. Let us examine the effect of a permeable partition on instability of this kind.

Figure 2 gives the results of a calculation of Gr_m (along the axis of ordinates there is plotted the value of $A = GrPr^{1/2}$, having an asymptote with large values of Pr) and the corresponding wave number k_m of the perturbations [curves 1 and 2) $\alpha = 0$; curves 3, 4) $\alpha = 1000$; curves 5 and 6) without a partition], as well as of the phase velocity $c = \lambda_i/kGr$ (curve 7).

For any given resistance of the partition, vibrational instability appears at $Pr_* = 8$, i.e., in this region of Prandtl numbers there is destabilization of the steady-state flow. An absolutely permeable partition ($\alpha = 0$) has a destabilizing effect on convective flow in

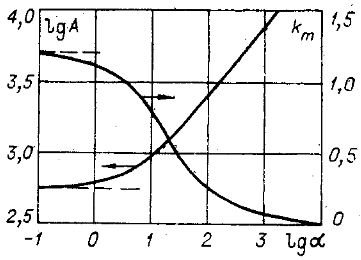

Fig. 3

in the whole region of Prandtl numbers; with a rise in Pr, this effect decreases. A partition with a large resistance (α = 1000), starting from Pr = 11.5, stabilizes the flow strongly.

The critical wave numbers of vibrational perturbations and their phase velocity, as can be seen in Fig. 2, rise rapidly with an increase in the Prandtl number and are stabilized in the region of large values of Pr. The value of the phase velocity of the critical perturbations is practically independent of the resistance of the partition.

In the flow under consideration, with a permeable partition, as for other kinds of convective motion in a layer of liquid, the limit of stability with respect to vibrational perturbations is lowered with a rise in the Prandtl number, and for large values of Pr the law $Gr_m \sim Pr^{-1/2}$ holds. The asymptotic behavior of the minimal Grashof number as $Pr \to \infty$ was investigated by the method of expansion of the solution with respect to the small parameter $Pr^{-1/2}$ [6]. The results of the calculations are given in Fig. 3. As can be seen, with an increase in the resistance of the partition, the stability of the flow increases. For $\alpha >$ 100, $Gr_m = 268\ (\alpha/Pr)^{1/2}$. Such a dependence of the minimal Grashof number on the resistance shows that an absolutely impermeable partition lowers this type of instability. With a rise in the resistance, the wave number of the most dangerous perturbation decreases monotonically while its phase velocity remains practically unchanged (c = 0.0678).

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